Proportionality Questions

Preface: 1) See the hand written notes on proportionality in the notes section for help. 2) Because I am teaching proportionality and not equation memorization, I will give you all of the necessary equations at the left of each question.

1. If the velocity of an object halves, by how much does the centripetal acceleration change? $$a_c = \frac{v^2}{r}$$

2. If the potential elastic energy of a spring is one fourth the original energy, by how much has the spring displacement changed? $$PE_{spring} = \frac{1}{2}kx^2$$

3. The length of a pendulum has doubled and the acceleration due to gravity is halved. By how much has the pendulum's period changed? $$T_{pendulum} = 2\pi\sqrt{\frac{\ell}{g}}$$

4. One of the masses has doubled and the distance between the masses has halved. How much has the force between them changed? $$F_g = G\frac{m_1 m_2}{r^2}$$

5. A new conductor has triple the resistivity and half the length of a different conductor. How does the cross-sectional area of the new conductor compare to the first? $$R = \rho\frac{\ell}{A}$$

6. The kinetic energy is reduced to 1/3 as much as the same time the mass is reduced to 1/12 as much. By how much has the velocity changed? $$KE = \frac{1}{2}mv^2$$

7. A new planet is found. It has 6 times the mass and a 2 times the gravitational field of the earth. What is the new planet's radius compared to the earth's? $$g = G\frac{M}{r^2}$$

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1. If the velocity of an object halves, by how much does the centripetal acceleration change? $a_c = \frac{v^2}{r}$

$a \propto v^2$

so $a' = (\Delta v)^2 a = (\frac{1}{2})^2 a = \frac{a}{4}$

2. If the potential elastic energy of a spring is one fourth the original energy, by how much has the spring displacement changed? $PE_{spring} = \frac{1}{2}kx^2$

solve for x, first

$x^2 = \frac{2(PE)}{k}$ $x = \sqrt{\frac{2(PE)}{k}}$ $x \propto \sqrt{PE}$ $x' = (\sqrt{\Delta PE})x = (\sqrt{\frac{1}{4}})x = \frac{x}{2}$

3. The length of a pendulum has doubled and the acceleration due to gravity is halved. By how much has the pendulum's period changed? $T_{pendulum} = 2\pi\sqrt{\frac{\ell}{g}}$

$T \propto \sqrt{\frac{\ell}{g}}$ $T' = \left(\sqrt{\frac{\Delta \ell}{\Delta g}}\right)T = \left(\sqrt{\frac{2}{1/2}}\right)T = \sqrt{4}\,T = 2T$

4. One of the masses has doubled and the distance between the masses has halved. How much has the force between them changed? $F_g = G\frac{m_1 m_2}{r^2}$

$F \propto \frac{m}{r^2}$ so $F' = \left(\frac{\Delta m}{(\Delta r)^2}\right)F = \frac{2}{(1/2)^2}F = \frac{2}{1/4}F = 8F$

5. A new conductor has triple the resistivity half the length of a different conductor. How does the cross-sectional area of the new conductor compare to the first? $R = \rho\frac{\ell}{A}$

$A = \frac{\rho \ell}{R}$ so $A \propto \frac{\ell}{R}$ so $A' = \left(\frac{\Delta \ell}{\Delta R}\right)A = \frac{1/2}{3}A = \frac{1}{2}\frac{1}{3}A = \frac{A}{6}$

$\frac{(1/2)}{(3/1)} = \frac{1}{2}\frac{1}{3} = \frac{1}{6}$

6. The kinetic energy is reduced to 1/3 as much as the same time the mass is reduced to 1/12 as much. By how much has the velocity changed? $KE = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2KE}{m}}$ so $v \propto \sqrt{\frac{KE}{m}}$ and $v' = \left(\sqrt{\frac{\Delta KE}{\Delta m}}\right)v = \sqrt{\frac{1/3}{1/12}}\,v = \sqrt{\frac{1}{3}\frac{12}{1}}\,v$

$= \sqrt{4}\,v$

$= 2v$

7. A new planet is found. It has 6 times the mass and a 2 times the gravitational field of the earth. What is the new planet's radius compared to the earth's? $g = G\frac{M}{r^2}$

$r = \sqrt{\frac{GM}{g}}$ so $r \propto \sqrt{\frac{M}{g}}$ and $r' = \sqrt{\frac{\Delta M}{\Delta g}}\,(r)$

$r' = \sqrt{\frac{6}{2}}\,r = \sqrt{3}\,r$

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