Rotational and Linear Motion

The rotational quantities all have correlations in linear motion.

Linear Quantities		Tangential Quantities		Rotational Quantities
Displacement (how far it moves in a straight line)	**x (m)**	*Arc length* **s**	**θ (rad)**	Angular Displacement (*theta*) (how much of a circle moves; angle)
Velocity (how fast it moves in a straight line)	**v (m/s)**	*Tangential velocity* v_t	**ω (rad/s)**	Angular Velocity (*omega*) (how fast it turns)
Acceleration (how fast it speeds up in a straight line – or how much the velocity changes)	**a (m/s^2)**	*Tangential acceleration* a_t	**α (rad/s^2)**	Angular Acceleration (*alpha*) (how fast it speeds up in circle –or how much ω changes)
Time (elapsed time)	**t (sec)**		**t (sec)**	Time (elapsed time)

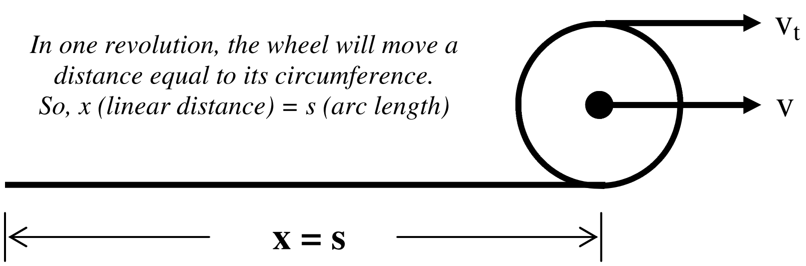
$$s = r\theta$$

$$v_t = r\omega$$

$$a_t = r\alpha$$

Tangential quantities allow you to translate between linear and rotational quantities. Tangential means "at this radius". If a merry-go-round has three rows of horses, the outside horse is going the fastest *tangentially*, (because it has the greatest radius) but they are all traveling at the same *angular* (rotational) velocity—they take the same amount of time to complete each circle.

In one revolution, the wheel will move a distance equal to its circumference. So, x (linear distance) = s (arc length)



$v_t = v$

The tangential velocity = the linear velocity of the wheel (linear velocity of the center of the wheel)

Kinetic Energy

When any object moves it has kinetic energy. When any object spins (or turns) it has rotational kinetic energy. If an object is both moving in a straight line and turning, its total kinetic energy is the sum of both.

$$E_{ktotal} = E_{klinear} + E_{krotational} = (1/2)mv^2 + (1/2)I\omega^2$$

For non-spinning objects $E_{krotational}$ is obviously zero.

Moment of Inertia

Inertia of a rotating object. For a point mass at a particular radius. $\mathbf{I = mR^2}$

Other I's: $I_{uniform\ disc} = (½)mR^2$; $I_{uniform\ sphere} = (2/5)mR^2$

Car rolling down a hill with rotating wheels.

$m_{car} = 2.0$ kg (wheels not included)

$m_{wheel} = .5$ kg (per wheel)

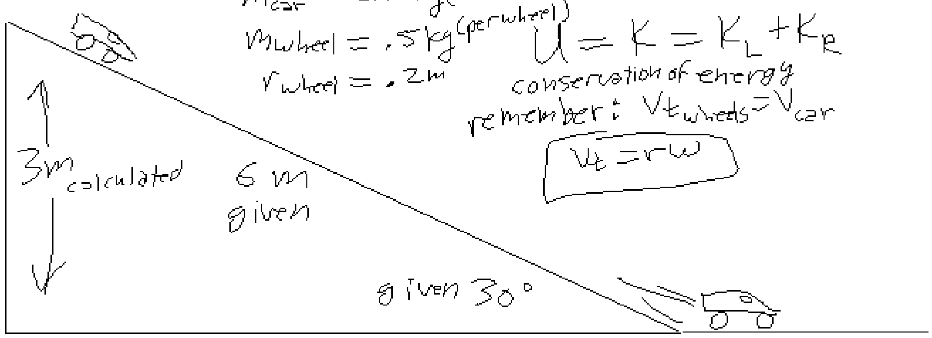
$r_{wheel} = .2$ m

$U = K = K_L + K_R$

conservation of energy

remember: $V_{t\,wheels} = V_{car}$

$\boxed{V_t = r\omega}$



$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

total — linear — wheels

$m =$

$h = 3$m

$m = 4$ kg $= 2 + 4(.5) = 4$ kg

$4(10)(3) = \frac{1}{2}(4)(v^2) + \frac{1}{2}\left(\frac{1}{2}\, 2\, [\cancel{R^2}]\, \frac{v_t^2}{\cancel{R^2}}\right)$ ($.5 \times 4$)

$I = 4$ (wheels)

$I = mR^2$ (point mass)

each wheel is a uniform disk,

$120 = 2v^2 + \underbrace{\frac{1}{2}v_t^2}_{\text{equal}}$ ☆

$120 = v^2(2 + \frac{1}{2})$

$120 = v^2(2.5)$

$v^2 = 48$

$v = 6.9$ m/s (see below if it slides)

$I = \frac{1}{2}mR^2$ unif. disk

$r\omega = v_t$

$\omega = \frac{v_t}{r}$

What if the wheels slip (no rotation)?

If $\mu = 0$ (slides down with no friction)

$U = K$ (no rot. K)

$mgh = \frac{1}{2}mv^2$

$10(3) = \frac{1}{2}v^2$

$30(2) = v^2 = 60$

$v = 7.7$ m/s

Notice v is faster if it slides. So part of the energy goes into rotating the tires.