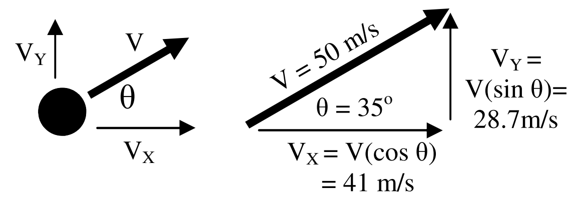
Projectile Motion Walk Thru—Ground to Ground

Background: An object launched into the air is a projectile. You should know that it comes down due to gravity, so its acceleration in the y-direction (its vertical acceleration) is $-9.8\ m/s^2$. You should also know that the acceleration in the x-direction = $0\ m/s^2$.

Ex 1: A projectile is launched at 35° going 50 m/s. It is launched from the ground and lands back on the ground. Calculate the time in the air and how far away it lands (known as its "range").

Step 1: Since the acceleration is only vertical, you have to work in the vertical and horizontal directions independently, so calculate Vxi (initial x-velocity) and Vyi (initial y-velocity).



Step 2: Write down everything you know (all the variables) in both directions (x and y).

<u>y-direction:</u>

$a_y = -9.8\ m/s^2$ (*freefall*)
Vi = Vsinθ = 28.7m/s (*see step 1*)
Vf = –Vi = –28.7m/s (*if gnd to gnd*)
Δy = 0m (*if gnd to gnd*)
t_y = ____

<u>x-direction:</u>

$a_x = 0\ m/s^2$ (*gravity is vertical only*)
So, S = D/T
Vi = Vcosθ = 41 m/s (*see step 1*)
Vf = Vi = 41 m/s (*since a = 0*)
Δx = ____
$t_x = t_y$ = ____

Step 3: From what you are given (your variables) solve for what you can.

<u>y-direction:</u>

$a_y = -9.8\ m/s^2$ (*freefall*)
Vyi = Vsinθ = 28.7m/s (*see step 1*)
Vyf = –Vyi = –28.7m/s (*if gnd to gnd*)
Δy = 0m (*if gnd to gnd*)
t_y = ____

We have all of the y variables, so we can solve for time.

<u>x-direction:</u>

$a_x = 0\ m/s^2$ (gravity is vertical only)
So, S = D/T or Vx = Δx/t (*since a = 0*)
Vxi = Vcosθ = 28.7m/s (*see step 1*)
Vxf = Vxi = 41 m/s (*since a = 0*)
Δx = D = ST = v_xt= ____
$t_x = t_y$ = ____

Can't solve for Δx or time. Need 1 more variable.

$$\Delta y = \frac{1}{2}(v_i + v_f)t$$
$$v_f = v_i + at$$
$$\Delta y = v_i t + \frac{1}{2}a(t)^2$$
$$\Delta y = v_f t - \frac{1}{2}a(t)^2$$
$$v_f^2 = v_i^2 + 2a\Delta y$$

Since we have all of the y-direction variables, we can use any of the equations (except the last one, since it doesn't have "t" in it). Don't choose the t^2 ones, since you would need the quadratic equation. If you use the 1st one v_i and v_f cancel. So use the 2nd one.

Vf = Vi + at
– 28.7 = 28.7 + – 9.8 t
–57.4 = –9.8t
t = 5.8 sec

Step 4: Now that you know ty, put it into your y-direction variables AND, since tx and ty are the same (it stops moving horizontally when it stops vertically), put it into the x-direction, too. Solve for x, now that you have time.

<u>y-direction:</u>

$a_y = -9.8\ m/s^2$ (*freefall*)
Vyi = Vsinθ = 28.7m/s (*see step 1*)
Vyf = –Vyi = –28.7m/s (*if gnd to gnd*)
Δy = 0m (*if gnd to gnd*)
t_y = **<u>5.8 sec</u>**

<u>x-direction:</u>

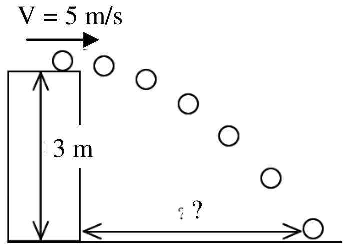
$a_x = 0\ m/s^2$ (gravity is vertical only)
So, S = D/T or Vx = Δx/t (*since a = 0*)
Vxi = Vcosθ = 41 m/s (*see step 1*)
Vxf = Vxi = 41 m/s (*since a = 0*)
Δx = D = ST = v_xt= ____
$t_x = t_y$ = 5.8 sec

Now we can solve for Δx

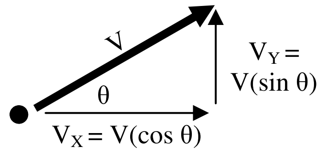
Δx = v_xt = 41(5.8)
<u>Δx = 238 m</u>

Projectile Motion Walk Thru—Horizontal Launch

Ex 2: A projectile is launched horizontally from 3 m up with an initial velocity of 5 m/s. Calculate its range (how far away it lands).



Step 1: Since the acceleration is only vertical, you have to work in the vertical and horizontal directions independently, so calculate Vxi (initial x-velocity) and Vyi (initial y-velocity).

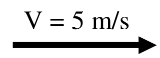


The x-velocity can always be calculated with cosine and the y-velocity with sine. A horizontally launched projectile has an angle of 0º, so:

Vy = 5sin0º = 0 m/s

Vx = 5cos0º = 5 m/s

Vx and Vy should also be obvious, since it is launched horizontally. It has no initial y-velocity, so Vyi = 0 m/s.



Step 2: Write down everything you know (all the variables) in both directions (x and y).

y-direction:

a_y = –9.8 m/s^2 (*freefall*)
Vi = Vsinθ = 0 m/s (*see step 1*)
Vf = ____
Δy = – 3 m (*it drops 3 m*)
t_y = ____

x-direction:

a_x = 0 m/s^2 (*gravity is vertical only*)
So, S = D/T and D = ST
Vi = Vcosθ = 5 m/s (*see step 1*)
Vf = Vi = 5 m/s (*since a = 0*)
Δx = ____
t_x = t_y = ____

Step 3: From what you are given (your variables) solve for what you can.

We could solve for Vf and t, but we don't need Vf. We do need time for the x-direction, though.

y-direction:

a_y = –9.8 m/s^2 (*freefall*)
Vi = Vsinθ = 0 m/s (*see step 1*)
Vf = ____
Δy = – 3 m (*it drops 3 m*)
t_y = ____

x-direction:

a_x = 0 m/s^2 (*gravity is vertical only*)
So, S = D/T and D = ST
Vi = Vcosθ = 5 m/s (*see step 1*)
Vf = Vi = 5 m/s (*since a = 0*)
Δx = D = ST
t_x = t_y = ____

If we had time, we could solve for Δx. So go to the y-direction.

$$\Delta y = \frac{1}{2}(v_i + v_f)t$$
$$v_f = v_i + at$$
$$\Delta y = v_i t + \frac{1}{2}a(t)^2$$
$$\Delta y = v_f t - \frac{1}{2}a(t)^2$$
$$v_f^2 = v_i^2 + 2a\Delta y$$

Again, many of you calculate Vf because you think you have to. You don't. The third equation doesn't use Vf, so let's try that one.

$$\Delta y = v_i t + \frac{1}{2}a(t)^2$$
$$-3 = 0(t) + \frac{1}{2}(-9.8)t^2 \quad \leftarrow \textit{0 times t = 0}$$
$$-3 = \frac{1}{2}(-9.8)t^2 \quad \leftarrow \textit{Only t is squared}$$
$$-3 = -4.9t^2$$
$$t^2 = -3/-4.9 = 0.612 \quad \leftarrow \textit{Don't forget to take the square root.}$$
$$t = \sqrt{0.612} = 0.78\ \text{sec}$$

Step 4: Now that you know ty, put it into your y-direction variables AND, since tx and ty are the same (it stops moving horizontally when it stops vertically), put it into the x-direction, too. Solve for x, now that you have time.

y-direction:

a_y = –9.8 m/s^2 (*freefall*)
Vi = Vsinθ = 0 m/s (*see step 1*)
Vf = ____
Δy = – 3 m (*it drops 3 m*)
t_y = **0.78 sec**

x-direction:

a_x = 0 m/s^2 (*gravity is vertical only*)
So, S = D/T and D = ST
Vi = Vcosθ = 5 m/s (*see step 1*)
Vf = Vi = 5 m/s (*since a = 0*)
Δx = D = ST
t_x = t_y = **0.78 sec**

Now we can solve for Δx

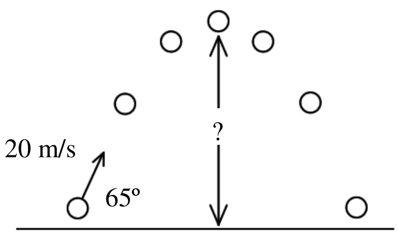
Δx = v_xt = 5(0.78)

Δx = 3.91 m

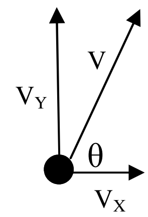
And we never needed V_f in the y-direction.

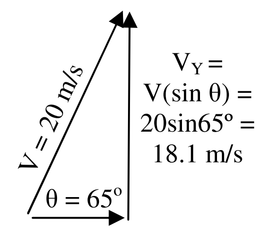
Projectile Motion Walk Thru—How High?

Ex 3: A projectile is launched 20 m/s at 65º. How high does it go?



Step 1: Since the acceleration is only vertical, you have to work in the vertical and horizontal directions independently. And since "How High?" is a vertical question, Vx is irrelevant, so just calculate Vyi.





Step 2: Write down everything you know (all the variables) in both directions (x and y).

y-direction:

$a_y = -9.8\ m/s^2$ (*freefall*)
Vi = Vsinθ = 18.1 m/s (*see step 1*)
Vf = 0 m/s (*at the top*)
Δy = ____ (*what we need)*
t_y = ____ (*don't need*)

x-direction:

Irrelevant, since "How High" is a vertical question only.

Step 3: From what you are given (your variables) solve for what you can.

We could solve for t, but we don't need it. We only need Δy.

y-direction:

$a_y = -9.8\ m/s^2$ (*freefall*)
Vi = Vsinθ = 18.1 m/s (*see step 1*)
Vf = 0 m/s (*at the top*)
Δy = ____ (*what we need)*
t_y = ____ (*don't need*)

$$\Delta y = \frac{1}{2}(v_i + v_f)t$$
$$v_f = v_i + at$$
$$\Delta y = v_i t + \frac{1}{2}a(t)^2$$
$$\Delta y = v_f t - \frac{1}{2}a(t)^2$$
$$v_f^2 = v_i^2 + 2a\Delta y$$

Notice that the last equation does not have t in it AND it has all of our other variables.

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$0 = (18.1)^2 + 2(-9.8)\Delta y$$
$$0 = 327.61 - 19.6\Delta y$$

← *Don't subtract. –19.6 is multiplied to Δy*

$$-327.61 = -19.6\Delta y$$
$$\Delta y = -327.61 / -19.6$$
$$\Delta y = 16.7m$$

Extension: Now that you have the highest point, you could find the time and then the x-direction position of the top of the arch. You will need t, though, first.

y-direction:

$a_y = -9.8\ m/s^2$ (*freefall*)
Vi = Vsinθ = 18.1 m/s (*see step 1*)
Vf = 0 m/s (*at the top*)
Δy = 16.7 m (*from step 3)*
t_y = ____ (*now needed for Δx*)

x-direction:

$a_x = 0\ m/s^2$ (*gravity is vertical only*)
So, S = D/T and D = ST
Vi = Vcosθ = 20cos65º = 8.45 m/s (*see step 1*)
Vf = Vi = 5 m/s (*since a = 0*)
Δx = D = ST
$t_x = t_y$ = **<u>1.8 sec</u>**

Vf = Vi + at
0 = 18.1 + – 9.8 t
–18.1 = –9.8t
t = 1.8 sec

Now we can solve for Δx

$\Delta x = v_x t = 8.45(1.8)$
Δx = 15.2 m

So the top point of this projectile 16.7 m up and 15.2 m from the starting point.