Harmonics on Strings and in Pipes

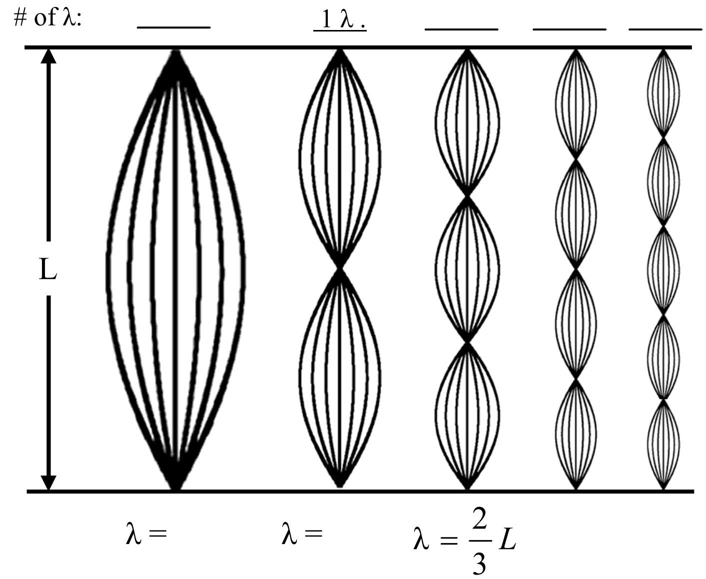
We will use the Socratic Method to derive equations for harmonics on strings and in pipes. "Sit and learn, Aristotle."

Harmonics on Strings

1) Fill in the equation at the right for how many antinodes (AN) equals 1 λ.
2) How many AN does the fundamental have?

$$\_\_\_\_\_ \ AN = \lambda$$

3) Above the diagram, write how many wavelength each harmonic is (*H_2 is already done*).
4) So, how many L's is the wavelength of the fundamental?



5) Under each harmonic, write its wavelength in terms of "L". (*H_3 is already done*).
6) You probably have the wavelength of the fundamental as a whole number, not a fraction. Make it a fraction, over 1.
7) Notice the denominators of harmonics 1, 3, and 5. You should see a pattern. Change the fractions of the other harmonics (2 and 4) so that the pattern is the same for all of them.
8) So, these changing denominators are the number of:

9) Calling the denominator "n" (the number of antinodes). Now write a formula for finding the wavelength of a particular harmonic on a fixed string of length "L".

$$\lambda_{\text{string}} =$$

10) * On a 2.5 m string, what is the wavelength of the 8th harmonic?

11) If a mechanical vibrator is vibrating 460 times per second to produce the 8th harmonic you just found, find the speed of the wave in the string.

One other thing to notice: on a string it is possible to have every harmonic. There are no "missing harmonics".

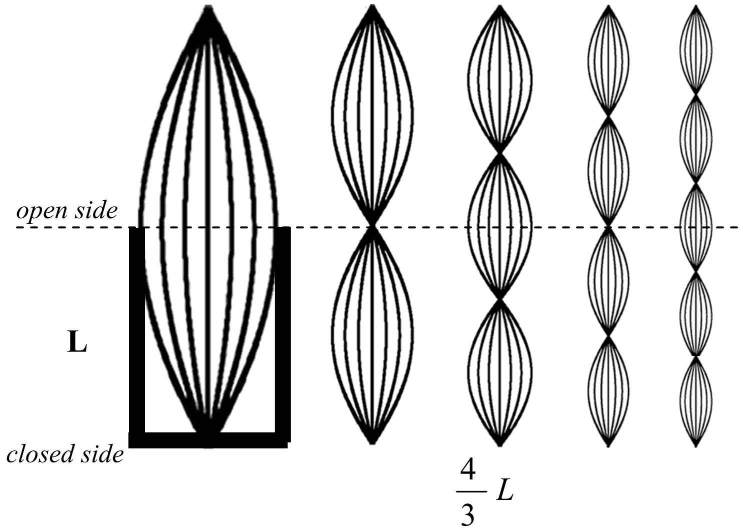
Transfer of Waves Between Mediums

The speed of the wave is different in different mediums, so the speed of the wave is different in the string than in air.

12) The speed of sound in air varies between 330 Hz and 444 Hz. Let's use $V_{sound} = 340$ m/s.
13) If you tighten the string, will the wavelength of the harmonics change (is the string longer)?
14) * When I tighten a string on my guitar what does change (music-wise)?
15) When the string vibrates, it beats against the air, making the air molecules vibrate, too. Every time the string pushes, the air moves, too. So, what is the same in air as in a string: the wavelength of the vibration or the frequency of the vibration?

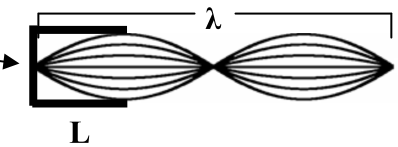
16) What frequency sound will we hear from the string in Q11?
17) Is this audible to us (*see "Sound" notes*)?
18) * What would be the wavelength of that sound in air? (*You have speed and frequency…)*

Harmonics in Closed Pipes



19) At the open end of a pipe is there a node or antinode?
20) At the closed end of a pipe is there a node or antinode?
So, there MUST BE a node at the closed end and the center of an antinode at the open end for a harmonic to exist.
21) On the graphic at the right, let's pretend we have a pipe closed at one end, as shown on the fundamental. The length of the pipe is "L".
22) Draw this **same length pipe** around each of the harmonics. The open end of each pipe will stop at the dotted line.
23) Which harmonics can be produced in this pipe (*which have an antinode at the open end*)?
24) So, for a pipe closed on one end (a closed pipe), give the sequence of what harmonics can be present. n can only =

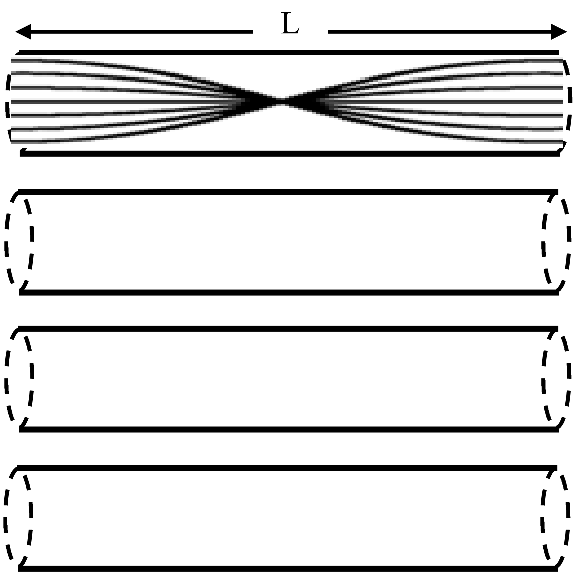
25) Since 2 AN = 1λ, then the first harmonic in this diagram, ______ λ = L *(see graphic)*.



26) So λ = ______L. Write this under H_1.
27) Use the same process as for the first harmonic, write the wavelengths under the harmonics (*even the ones that can't really exist*). H_2 and H_4 should be obvious.
28) Remembering Q7-9: there must be a pattern that includes "n" sequentially. Find it.
29) * At the right, write the equation for the wavelength of a harmonic in a closed pipe:
30) * But which "n's" are possible?
31) * Calculate the wavelength of the third harmonic in a 60 cm long closed pipe (*remember to convert to m*).

$\lambda_{\text{closed pipe}} =$

Harmonics in Open Pipes



32) Remembering the rule about nodes and antinodes, what must be present at the end of a pipe that is open?
33) In order for a pipe to make a sound, it must have a standing wave in it, meaning it must have at least 1 node and 1 antinode (or more).
34) For each successive harmonic the number of antinodes goes up by 1.
35) For each successive harmonic the number of nodes goes up by ____.
36) * In the four open pipes at the left, draw the first four harmonics (*number 1 is done for you*). The nodes will be equally spaced and the center of an antinode must be at each end.
37) On a string you count the number of _______________ to figure out which harmonic it is. For an open pipe, you count the number of ______________ to figure out which harmonic it is.
38) * Which harmonics can be present in an open pipe (*which "n's" are present*)?

39) * Using the same logic, figure out the sequence and give the equation for λ in an open pipe. *(Hint: the drawing above will help.)*

$\lambda_{\text{open pipe}} =$

40) * What is the wavelength of the 3rd harmonic in a 80 cm long open pipe?

Putting it all together:

41) A **string** is 1.5 m long and produces a note that has a frequency of 150 Hz when plucked. This is H_1, the fundamental.
   A. As a string the $\lambda_{fundamental}$ = __ L.
   B. * So, $\lambda_{fundamental}$ =

   C. Calculate the speed of the wave on the string.

   D. * Give the first 3 possible harmonics on this string.

   E. What part of the sound will be the same in air?
   F. * If the speed of sound in air is 343 m/s, what is the wavelength of the note in the air?

42) An **open pipe** 3 m long produces a 56 Hz sound as its natural frequency (fundamental).
   A. Since it is an open pipe, the $\lambda_{fundamental}$ = __ L.
   B. * Calculate the wavelength of the fundamental.

   C. In a pipe it is actually air that is vibrating, so find the speed of the wave in the pipe (*which is the speed of sound in air*).

   D. Give the first 3 possible harmonics on this string.

43) An 40 cm pipe is closed at one end. When struck it naturally produces a 206 Hz sound (*its natural frequency, the fundamental*).
   A. Since it is an closed pipe, the $\lambda_{fundamental}$ = __ L.
   B. * So, $\lambda_{fundamental}$ =

   C. Calculate the speed of sound of the air in the pipe.

   D. * Give the first 3 possible harmonics on this string.

All of the above are pretty simple if you remember that for a string or open pipe $\lambda_{fundamental}$ = 2L and for a closed pipe $\lambda_{fundamental}$ = 4L. And each of the three above examples work with the fundamentals only. Here's how you deal with other examples, easily.

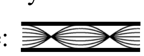
44) * A **closed pipe** is 20 cm long. The third harmonic on the pipe is 1275 Hz. Calculate the velocity of air in the pipe.
   A. You need the wavelength and frequency of one particular harmonic on the pipe. So, calculate the frequency and wavelength of the fundamental.

   B. You can now use the wave equation to calculate the wave speed.

45) If the speed of sound in air is 336 m/s. An open pipe makes a fourth harmonic of 480 Hz. What is the length of the pipe?
   A. Calculate the frequency of the fundamental.

   B. Calculate the wavelength of the fundamental.

   C. Knowing that $\lambda_{fundamental\ for\ an\ open\ pipe}$ = ____ L, calculate the length of the pipe.

* - Q10) $\lambda_1 = 2.5(2) = 5.0$ m, so $\lambda_8 = 5.0/8 = 0.625$ m. OR $\lambda_8 = 2(2.5)/8$ Q14) f Q18) 0.739 m Q29) $\lambda = 4L/n$ Q30) only odds (evens have a node at the end) Q31) $\lambda_1 = 4L = 4(.6) = 2.4$ m, $\lambda_3 = 2.4/3 = .8$ m (and H_2 doesn't exist). Q36) H_2 looks like: 
Q38) all of them. Q39) same as for strings: $\lambda = 2L/n$ Q40) open so: $\lambda_1 = 2L = 2(.8) = 1.6$m; $\lambda_3 = 1.6/3 = 0.53$ m
Q41) B: 3m; D: $f_1 = 150$ Hz; $f_2 = 300$ Hz, etc. F: 2.9 m/s; Q42) B: 6 m; D: all harmonics possible, so just multiply f_1 by 1,2,3
Q43) B: 1.6 m; D: only odd harmonics this time (close pipe). Q44) A: $\lambda_1 = 4(.2) = 0.8$ m; $f_1 = 1275/3 = 425$ Hz B: v = 340 m/s

